

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM1

Branch: B.Tech (CE)

Semester: 4

Date: 15/04/2019

Time: 02:30 To 05:30

Marks: 70

### Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
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- Q-1 Attempt the following questions: (14)**
- a) Find the least and greatest element in the poset  $\langle N, D \rangle$ , if they exist. (02)
  - b) Define: Poset, Pseudo Graph (02)
  - c) State Pigeonhole principle. (02)
  - d) Find the atom and anti-atom of  $\langle S_{60}, D \rangle$ . (02)
  - e) Prove that  $(ab + ab')a'b' = 0$ . (02)
  - f)  $(Z_{11}, +_{11})$  is cyclic group.- True or False? (01)
  - g) How many edges are there in a graph with 7 vertices each of degree 4? (01)
  - h) Define: Difference of Fuzzy set (02)

### Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions. (14)**
- a) State and prove Stone's representation theorem. (10)
  - b) State Distributive law for fuzzy subsets and prove any one. (04)
- Q-3 Attempt all questions (14)**
- a) Show that  $\{0, 2, 4, 6\}$  is a subgroup of  $(Z_8, +_8)$ , where  $+_8$  is addition modulo 8. (05)
  - b) Prove that  $\langle S_{42}, D \rangle$  is a complemented lattice and also draw the Hasse diagram of it. (05)
  - c) Prove that  $\langle S_6, D \rangle$  is a sub lattice of  $\langle S_{30}, D \rangle$ . (04)
- Q-4 Attempt all questions (14)**
- a) Show that the set  $Q \setminus \{-1\}$  is an abelian group with respect to the binary operation  $a * b = a + b + ab$ , for all  $a, b \in G$ . (07)



- b) Let  $E = \{a, b, c\}$ ,  $\underline{A} = \{(a, 0.4), (b, 0.7), (c, 0.6)\}$ ,  $\underline{B} = \{(a, 0.8), (b, 0.2), (c, 0.5)\}$  then (07)  
 find the following:  
 1)  $\underline{A} \cup \underline{B}$  2)  $\underline{A} \cdot \underline{B}$  3)  $\underline{A} \hat{+} \underline{B}$  4)  $\underline{A} - \underline{B}$  5)  $\underline{A} \cap \underline{B}$  6)  $\underline{A}'$  7)  $\underline{B}'$

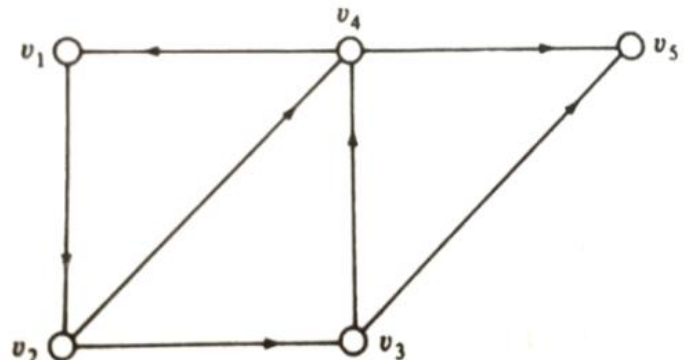
**Q-5 Attempt all questions (14)**

- a) For a lattice  $\langle P(\{a, b, c\}), \subseteq \rangle$ , answer the following questions: (07)  
 i) Find cover of each element and draw the Hasse diagram.  
 ii) Find lower bound, upper bound, greatest lower bound, least upper bound of  $A = \{\{a, b\}\}$ .  
 iii) Find the least and greatest element of it.  
 b) Let  $\langle L, \leq \rangle$  be a lattice  $a, b \in L$  then prove that (07)  
 i)  $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$  ii)  $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

**Q-6 Attempt all questions (14)**

- a) i) Draw the graph represented by given adjacency matrix (05)  

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$
  
 ii) Write the adjacency matrix from the given digraph.

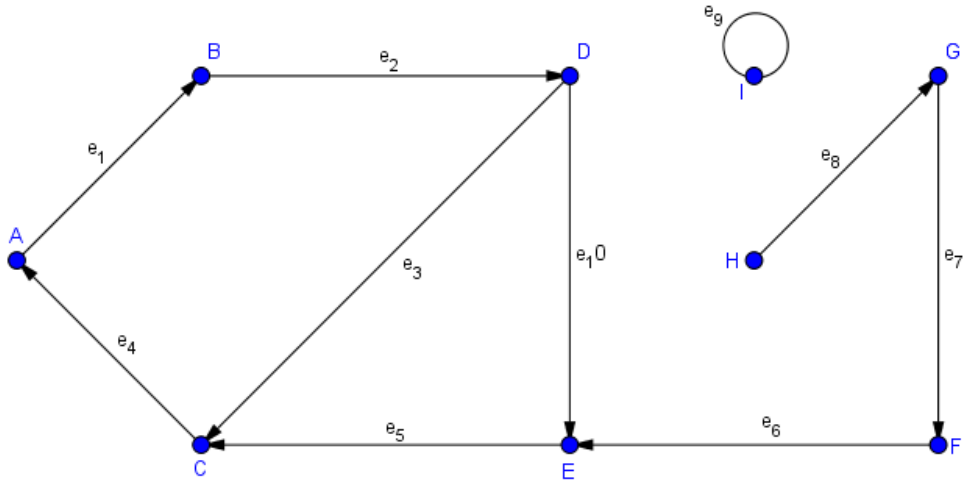


- b) State and prove Lagrange's theorem. (05)  
 c) By using mathematical induction prove that  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ . (04)

**Q-7 Attempt all questions. (14)**

- a) Obtain the sum of product canonical form of the Boolean expression in three variables  $\alpha(x, y, z) = (x \oplus y)' \oplus z$ . (05)  
 b) Prove that  $(Z_6^*, \times_6)$  is a group. Is it commutative? (05)

c) Find all node base of the following diagram shown in the figure. (04)



**Q-8 Attempt all questions.** (14)

a) Define tree and draw a directed tree from following and also find the representation of (07)

binary tree.  $(v_0(v_1(v_2)(v_3(v_4)(v_5)))(v_6(v_7(v_8))(v_9)(v_{10})))$

b) Do as directed: (07)

1) Translate the following in your own words.

$A(x)$ :  $x$  is a whale,  $B(x)$ :  $x$  is a fish,  $C(x)$ :  $x$  lives in water.

i)  $(\exists x)(B(x) \wedge \sim A(x))$       ii)  $(\forall x)(A(x) \vee C(x)) \Rightarrow B(x)$

2) Solve the recurrence relation  $a_n = 5a_{n-1} - 6a_{n-2}$ ,  $n \geq 2$ ;  $a_0 = 1$ ,  $a_1 = 2$ .

